Exam. Code : 103205
Subject Code : 1202

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—I

(Dynamics)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any ***five*** questions in all choosing at least ***two*** from each section. All questions carry equal marks.

SECTION—A

1. A particle moving with uniform acceleration in a straight line passes points A, B and C. If AB = BC = b and if time from A to B is t_1, B to C is t_2, prove that the acceleration is $\frac{2b(t_1 - t_2)}{t_1 t_2 (t_1 + t_2)}$. 10

2. A ball is dropped from the top of a tower h meters high and at the same moment another ball is projected upwards from the bottom. They meet when the upper one has described $\frac{1}{k}$th of the total distance. Show that their speeds when they meet are in the ratio 2 : (k – 2) and that the initial velocity of the lower ball is $\frac{1}{2}\sqrt{kgh}$. 10

3. A mass m_1 hanging vertically is connected to another mass m_2 placed on a smooth inclined plane of inclination α by means of a light inelastic string passing over a smooth pulley fixed at the top of the plane. The system is released from rest, discuss the motion and find the pressure on the pulley. 10

4. A particle, moving in a straight line is subjected to a retardation of kv^n per unit mass, where v is the speed at time t. Show that if, $n < 1$, the particle will come to rest at a distance $\frac{u^{2-n}}{k(2-n)}$ from the point of projection at time $t = \frac{u^{1-n}}{k(1-n)}$, where u is initial speed.

   What happens when :

   (i) $1 < n < 2$

   (ii) $n > 2$ 10

5. (a) Define SHM. Prove that simple harmonic motion is periodic and its period is independent of the amplitude.

   (b) A particle is moving between two points A and B in SHM. If the period of oscillation is 2π, show that the velocity at any point P is mean proportional between AP and BP. 5,5

SECTION—B

6. A particle of mass m is projected from a fixed point with velocity u in a direction making an angle $\alpha \left(\neq \frac{\pi}{2} \right)$ with the horizontal. Neglecting the air resistance, find its motion and show that its path is a parabola. 10

7. A particle is projected along the inner surface of a smooth vertical circle of radius r, its velocity at the lowest point being $\frac{1}{5}\sqrt{95\,r\,g}$. Show that it will leave the circle at angular distance $\cos^{-1}\left(\frac{3}{5}\right)$ from the highest point and its velocity then is $\frac{1}{5}\sqrt{15\,r\,g}$. 10

8. Define areal velocity. Prove that with usual notations, the area velocity of a particle moving along a plane curve is $\frac{1}{2}v\,p$. 10

9. (a) Define work and power. Discuss F.P.S. and M.K.S. system of units of work and power.
   (b) A particle of mass m falls from rest at a height h above the ground. Show that throughout the motion, the sum of kinetic and potential energies is constant. 4,6

10. A seconds pendulum was too long on a given day by a quantity a, it was then over corrected so as to become too short by a during the next day. Prove that if l is the correct length, then the number of minutes gained in two days are $1080\frac{a^2}{l^2}$ nearly. 10